Columbia Math Tournament Tiebreaker Round

1. Find all integer solutions to $x^3 + y^3 + z^3 - 3xyz = 5$
2. Suppose α, β are real roots of the polynomial $x^2 - rx + s$, with $(r, s) \in \mathbb{R}$. Define the following sequence:

$$a_1 = r$$

$$a_2 = r^2 - s$$

$$a_n = r * a_{n-1} - s * a_{n-2}$$

   Find an explicit formula for a_n in terms of α and β.

3. Call a lattice point $p \in \mathbb{R}^2$ *visible* from a point p' if there are no other lattice points on the line segment $\overline{pp'}$. Given a random lattice point, what is the probability it is visible from the origin?
4. Let p_i be the i^{th} prime number, prove for sufficiently large n:

$$\prod_{i=1}^{n} p_i > p_{n+1}^2$$